An ABC of PME

Rüdiger Stucke, PhD
Landmark Fellow

SL Capital Partners Research Fellow, Saïd Business School, University of Oxford
ruediger.stucke@sbs.ox.ac.uk

Barry E. Griffiths, PhD, CFA
Director of Quantitative Research
Barry.Griffiths@landmarkpartners.com
(860) 408-4154

Ian H. Charles, CFA
Partner
Ian.Charles@landmarkpartners.com
(860) 408-4146

Motivation

Institutional investors, fund managers, and consultants have been trying to compare private equity returns against public markets ever since the pioneering work of Long and Nickels on the Index Comparison Method, or ICM (Long and Nickels, 1995). However, those investors quickly get lost in a jungle of different approaches, each of which is aiming to avoid some perceived shortcomings of its predecessors and trying to give a better estimation of the generated return relative to a benchmark. Familiar names include ICM, PME+ (Rouvinez, 2003), mPME (Cambridge Associates, 2013), and KS-PME (Kaplan and Schoar, 2005). Our earlier contribution to this thicket was the method we now call *Direct Alpha* (Griffiths, 2009; Griffiths, 2010).

As a result, the practical problem facing investors is not that there is no way to compare the returns of illiquid and liquid assets, but that there are too many. The different methods produce results that are often similar, but sometimes very different. The original descriptions of the different methods do not lend themselves to straightforward comparison. To date, no one has produced a thorough analysis of the mathematical relationships and real-world behavior of the different PME techniques.

In this white paper we will show that although the original descriptions of the various PME methods seem quite different, they are closely related mathematically. We will also show that Direct Alpha is the simplest of these methods to calculate. Finally, we will demonstrate the relative numerical reliability of these methods over a large set of real-world cash flow data. A more complete discussion of these issues can be found in Gredil, Griffiths, and Stucke (2014).

The Direct Alpha Method and the KS-PME

The calculation of Direct Alpha is quite straightforward. There are two steps:

1. Find the future value (at final valuation time) of each contribution and distribution, according to the reference benchmark
2. Find the Internal Rate of Return (IRR) for that sequence of future-valued cash flows.

The future value of each cash flow at valuation time is just the actual cash flow, multiplied by the ratio of the benchmark at valuation time to the benchmark at the actual time of the cash flow. The future value of the NAV is, of course, unchanged. (We use future values for simplicity in exposition. Present values, or indeed values as of any one date, would serve as well.) Any benchmark can be used in the Direct Alpha calculations, but the economic meaning of the results depends very much on what index is used. The Direct Alpha result is really only

Landmark Partners

Connecticut
10 Mill Pond Lane
Simsbury CT 06070
(860) 651-9760

Massachusetts
265 Franklin Street, 18th Floor
Boston, MA 02110
(617) 556-3910

New York
681 Fifth Avenue, 14th Floor
New York, NY 10022
(212) 858-9760

United Kingdom
52 Jermyn Street
London, SW1Y 6LX
44 (20) 7343-4450

www.landmarkpartners.com

"alpha" when the reference index contains all of the systematic components of return. For more discussion, see Gredil, Griffiths, and Stucke (2014).

The Direct Alpha method produces a *rate of return* of outperformance. The result of the IRR is an arithmetic measure of outperformance, but this can be easily transformed into a log rate if desired. An informal derivation of the Direct Alpha method is presented in Appendix A. The details in a stochastic process setting, including confidence intervals on the estimator, are found in Griffiths (2009).

Many readers will recognize that the Direct Alpha method has much in common with Kaplan and Schoar's Public Market Equivalent, which we will refer to as KS-PME for convenience. The KS-PME is a relative outperformance *multiple*. Similar to Direct Alpha, there are two steps to computing the KS-PME:

1. Find the future value (at final valuation time) of each contribution and distribution, according to the reference benchmark
2. Find the TVPI (Total Value to Paid In) ratio for that sequence of future-valued cash flows.

Figure 1 presents a simple numerical example for the calculation of Direct Alpha and the KS-PME. The actual contributions (C) and distributions (D) of the PE fund are compounded by the returns of the public equity index up to Dec-31, 2010, and then combined with the final NAV to form the series of future values of net cash flows. For example, the future value of the contribution of 50 at Dec-31, 2005, is calculated by multiplying it with the return of the index over the period Dec-31, 2005 to Dec-31, 2010: 50 * 131 / 117 = 56. The IRR over this series of future values of the net cash flows yields the (arithmetic) Direct Alpha of the PE fund relative to the reference benchmark. At the same time, the TVPI ratio of the series of future values yields the KS-PME wealth ratio: (453+75) / 317.

	Actual Values					Future Values			
	C	D	NAV_{PE}	Net CF	Index	FV (C)	FV (D)	NAV_{PE}	FV (Net CF)
Dec-31, 2001	100	0	...	-100	100	131	0	...	-131
Dec-31, 2002	0	0	...	0	78	0	0	...	0
Dec-31, 2003	100	25	...	-75	100	130	33	...	-98
Dec-31, 2004	0	0	...	0	111	0	0	...	0
Dec-31, 2005	50	150	...	100	117	56	168	...	112
Dec-31, 2006	0	0	...	0	135	0	0	...	0
Dec-31, 2007	0	150	...	150	142	0	138	...	138
Dec-31, 2008	0	0	...	0	90	0	0	...	0
Dec-31, 2009	0	100	...	100	113	0	115	...	115
Dec-31, 2010	0	0	75	75	131	0	0	75	75
Total	250	425				317	453		
			IRR:	17.5%		**Direct Alpha (arithmetic):**			12.6%
			TVPI:	2.00				**KS-PME:**	1.67

Figure 1: Numerical example of the Direct Alpha approach using future values

In this example the Direct Alpha is 12.6%, representing the annualized rate of return (beyond the public equity returns) that is solely attributable to PE. Similarly, the KS-PME is 1.67, indicating that the final wealth is 1.67 times what would have been achieved by investing in the reference benchmark.

The underlying rationale of compounding all PE cash flows to the same single point in time is to 'remove' or 'neutralize' the impact of any changes in the public equity index from the series of actual PE cash flows. By doing so, the resulting capitalized net cash flows no longer 'contain' any changes of the index, but reflect only the sole value creation that is attributable to PE (i.e., the rate of return above or below the index returns).

As described above, it is critical to use the reference public equity index to capitalize all PE cash flows to the same single point in time. In line with the natural process of value creation, we have followed the perspective of future values above. However, it is equally possible to capitalize all PE cash flows (and the final NAV) by the index returns to any other point in time with the arithmetic alpha a remaining the same. For example, instead of future values one can equally follow the present value perspective.

Figure 2 shows how the actual contributions, distributions and the NAV_{PE} in our numerical example are transformed into their present values back to Dec-31, 2001. For example, the present value of the distribution of 150 at Dec-31, 2007, is calculated by dividing it by the return of the index over the period Dec-31, 2001 to Dec-31, 2007: 150 / (142 / 100) = 105.

As a result, the series of discounted net cash flows changes in nominal terms. However, the series of present values in Figure 2 and the series of future values in Figure 1 differ only by a single constant factor (1.31) and, hence, the 'relationship' of the net cash flows within each series remains unaffected. As a result, the arithmetic alpha remains the same, as does the KS-PME ratio.

While it is only a matter of taste whether to compound the actual PE cash flows to their future values, or to discount them to their present values, some practitioners may find the present value perspective more intuitive. It can be interpreted as 'removing the contribution' of the public equity returns from all of the subsequent PE cash flows following Dec-31, 2001. A small advantage of using future values is, however, that one can use the PE fund's NAV at face value and does not need to capitalize it to a different point in time.

	Actual Values					**Present Values**			
	C	**D**	**NAV_{PE}**	**Net CF**	**Index**	**PV (C)**	**PV (D)**	**PV (NAV_{PE})**	**PV (Net CF)**
Dec-31, 2001	100	0	...	-100	100	100	0	...	-100
Dec-31, 2002	0	0	...	0	78	0	0	...	0
Dec-31, 2003	100	25	...	-75	100	100	25	...	-75
Dec-31, 2004	0	0	...	0	111	0	0	...	0
Dec-31, 2005	50	150	...	100	117	43	129	...	86
Dec-31, 2006	0	0	...	0	135	0	0	...	0
Dec-31, 2007	0	150	...	150	142	0	105	...	105
Dec-31, 2008	0	0	...	0	90	0	0	...	0
Dec-31, 2009	0	100	...	100	113	0	88	...	88
Dec-31, 2010	0	0	75	75	131	0	0	57	57
Total	250	425				243	347		
			IRR:	17.5%				**Direct Alpha (arithmetic):**	12.6%
			TVPI:	2.00				**KS-PME:**	1.67

Figure 2: Numerical example of the Direct Alpha approach using present values

As we can see, the (arithmetic) Direct Alpha and the KS-PME are intimately related. Direct Alpha represents the market-adjusted equivalent to the traditional IRR of a PE fund. KS-PME, which is also directly derived from the future values of contributions and distributions as shown in Figure 1, represents the market-adjusted equivalent to the traditional TVPI.

	Rate of return	Total return
Absolute return	IRR	TVPI
Market-adjusted return	Direct Alpha	KS-PME

Figure 3: Relationship between the absolute and the market-adjusted performance measures

Direct Alpha and other Public Market Equivalent Approaches

As shown above, Direct Alpha is *directly* calculated based on the series of private equity cash flows that are capitalized by the benchmark returns to the same single point in time. In this respect, Direct Alpha differs from other approaches which calculate the rate of return of some hypothetical public equity portfolio (the 'public market equivalent'), and then *indirectly* estimate alpha as the spread between the IRR of the hypothetical public portfolio and the IRR of the original PE fund.

In Appendix B we give a more formal description of each of three well-known PME approaches, including their individual shortcomings. The main problem that ICM, PME+, and mPME have in common, though, is that is that they cannot – by definition – arrive at a PE fund's exact rate of return relative to the benchmark. This is due to the non-additive nature of compound rates which effectively prevents the calculation of the exact alpha in an *indirect* way (see Cauchy's functional equation). As a result, all of these PME approaches represent heuristic approximations by definition.

Figure 4 shows how Direct Alpha relates to the three PME approaches. We see that the heuristic approximations in both ICM/PME and PME+ require the analyst to do almost all of the calculations needed for the exact solution of Direct Alpha, but then require the analyst to also do additional calculations related to their hypothetical public market portfolios. While mPME also uses a hypothetical public market portfolio its construction is somewhat different conceptually, and it still requires more complex calculations to yield a less accurate result.

ICM/PME. ICM/PME builds a hypothetical public market portfolio by investing a PE fund's contributions into, and divesting a PE fund's distributions from, a public equity index to eventually arrive at a different residual value. There are five conceptual steps in the ICM/PME approach:

1. Find the future value (at final valuation time) of the contributions and distributions, according to the reference benchmark;
2. Find the NAV of the hypothetical public portfolio;
3. Find the IRR of the hypothetical public portfolio that has the PE fund's original contributions, the PE fund's original distributions, and the hypothetical NAV from Step 2;
4. Find the IRR of the PE fund;
5. Find the ICM/PME spread, which is the difference between the IRRs found in Steps 3 and 4.

Obviously the hypothetical public-market NAV in Step 2 can be expressed as the difference of the sum of the future values of all contributions (C) minus the sum of the future values of all distributions (D):

$$NAV_{ICM} = \sum FV(C) - \sum FV(D) \quad (1)$$

Note that these sums of future values are among the outputs of the Direct Alpha/KS-PME approach, as depicted in Figure 4. While this approach is intuitively appealing, using this hypothetical NAV can cause certain problems since the public portfolio does not liquidate as the PE fund does. Similarly, a negative NAV in the public portfolio by some point may effectively prevent the calculation of an ICM/PME IRR.

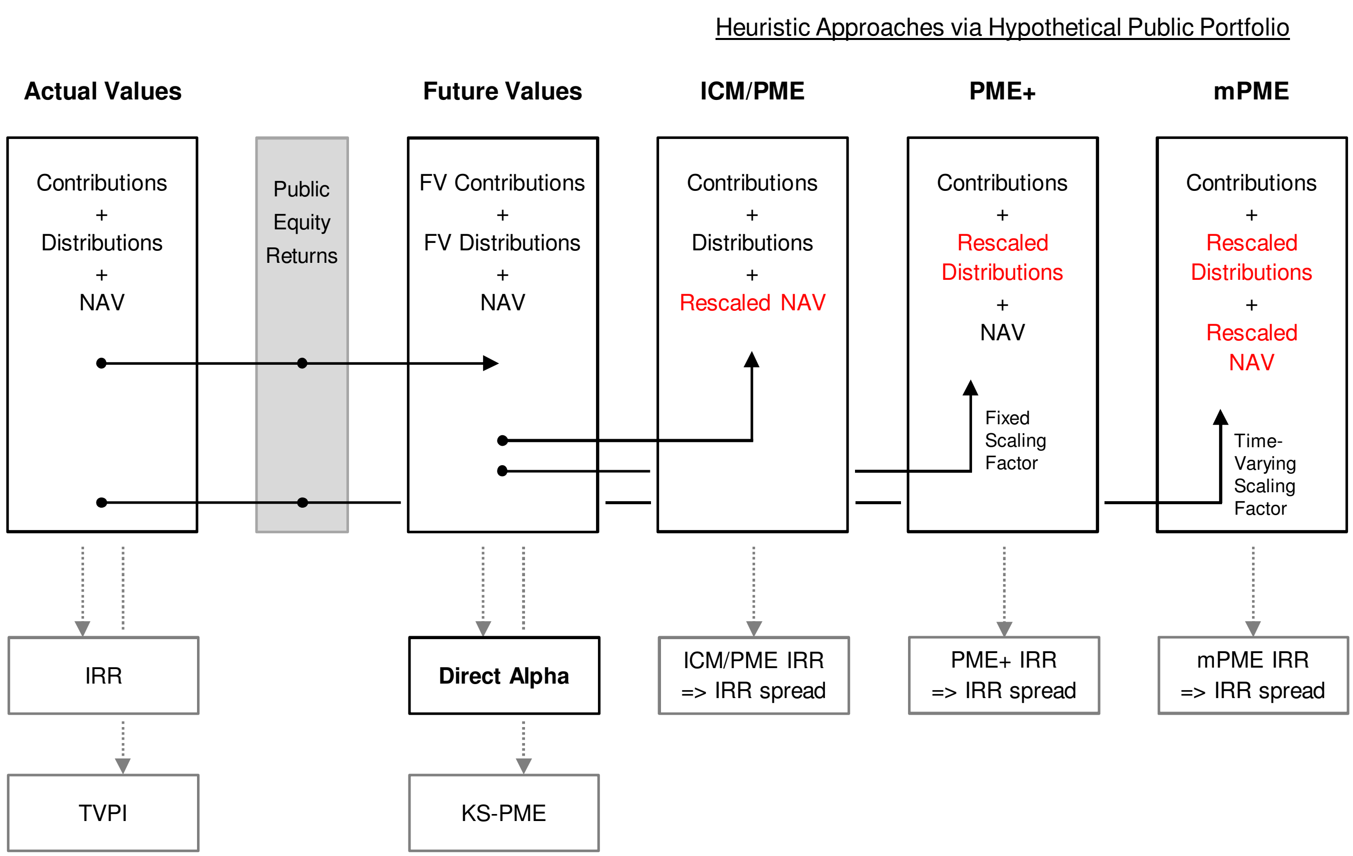

Figure 4: Conceptual relationship between Direct Alpha and the heuristic approaches

PME+. PME+ is very similar in concept to ICM/PME, relying on the same idea of solving for the IRR of a hypothetical public portfolio. However, PME+ is designed to avoid the problems that arise from the fact that the hypothetical public portfolio in ICM/PME does not liquidate. Instead, PME+ calls for rescaling the distribution sequence in order to keep the same NAV as the PE fund. Thus, when the PE fund liquidates, the hypothetical public portfolio in PME+ also liquidates. Just as with ICM/PME, there are five conceptual steps in the PME+ approach:

1. Find the future value (at final valuation time) of the contributions and distributions, according to the reference benchmark;
2. Find a constant scaling factor for the distributions that will maintain the PE fund's original NAV;

3. Find the IRR of the hypothetical public portfolio that has the PE fund's original contributions, the rescaled PE fund's distributions from Step 2, and the PE fund's original NAV;
4. Find the IRR of the PE fund;
5. Find the PME+ spread, which is the difference between the IRRs found in Steps 3 and 4.

Finding the constant scale factor in Step 2 of PME+ is actually very similar to finding the hypothetical NAV in Step 2 of ICM/PME. It can be easily seen that the PME+ scale factor, *s*, is governed by the future-value relationship in Equation 2:

$$NAV_{PE} = \sum FV(C) - s \cdot \sum FV(D) \quad (2)$$

The future-value relationship in Equation (2) that defines PME+ is very similar to the future-value relationship in Equation (1) that defines ICM/PME. Note once again that these sums of future values of cash flows are among the outputs of the Direct Alpha and KS-PME approach.

mPME. The newest of these methods, mPME once again draws on Long and Nickels' idea of finding the IRR of a hypothetical public portfolio. Like PME+, mPME forces the hypothetical public portfolio to liquidate at the same time as the PE fund. Unlike PME+, mPME does not apply a constant scale factor to the entire sequence of distributions. And unlike both ICM/PME and PME+, mPME cannot be expressed in terms of the future-value relations that are used in Direct Alpha. The conceptual steps in mPME are as follows:

1. Construct a hypothetical public portfolio in the following manner:
    a. At each time, the contributions are the same as for the PE fund;
    b. At each time, the distributions have the *same proportion to NAV* as for the PE fund;
    c. At each time, the rate of return is the same as in the reference benchmark;
2. Find the IRR of this hypothetical public portfolio;
3. Find the IRR of the PE fund;
4. Find the mPME spread, which is the difference between the IRRs found in Steps 2 and 3.

Due to the time-recursion that is used to construct the hypothetical public portfolio in Step 1, the NAV of the hypothetical public portfolio in mPME is generally not the NAV of the PE fund, as required by PME+. At the same time, the mPME cash distributions are not simply rescaled versions of the PE fund's actual cash distributions, as calculated in PME+. mPME does have the useful property that the hypothetical public portfolio is fully liquidated whenever the PE fund is fully liquidated. It shares this property with PME+, although it gets there by a different method of construction.

Direct Alpha versus PME IRR Spreads

While the Direct Alpha method generates robust results by construction,[1] we now examine the extent to which Direct Alpha generates more reliable results. To gauge the level of accuracy of the existing heuristics, we use a sample of 500 private equity and venture capital funds with vintage years from 1990 to 2007. For each fund we calculate the Direct Alpha relative to the S&P 500, as well as the ICM/PME IRR spread, the PME+ IRR spread, the mPME IRR spread, and an annualized version of the KS-PME as further explained below.

Figure 5 shows a series of scatter plots which contain the funds' Direct Alphas on the x-axis, and the corresponding IRR spreads of each approximation on the y-axis. We limit the scale of the axes to the interval [-20%, +20%], which contains

[1] Having applied the Direct Alpha method to about 4,000 private equity funds, we found that Direct Alpha could always be calculated if there was also a valid solution for the PE fund's IRR, which has been the case for 99.8% of all funds.

close to 90% of all observations. Identical values should line-up on the 45 degrees diagonal, i.e., the extent to which each observation deviates from the diagonal line quantifies the error of the approximation.

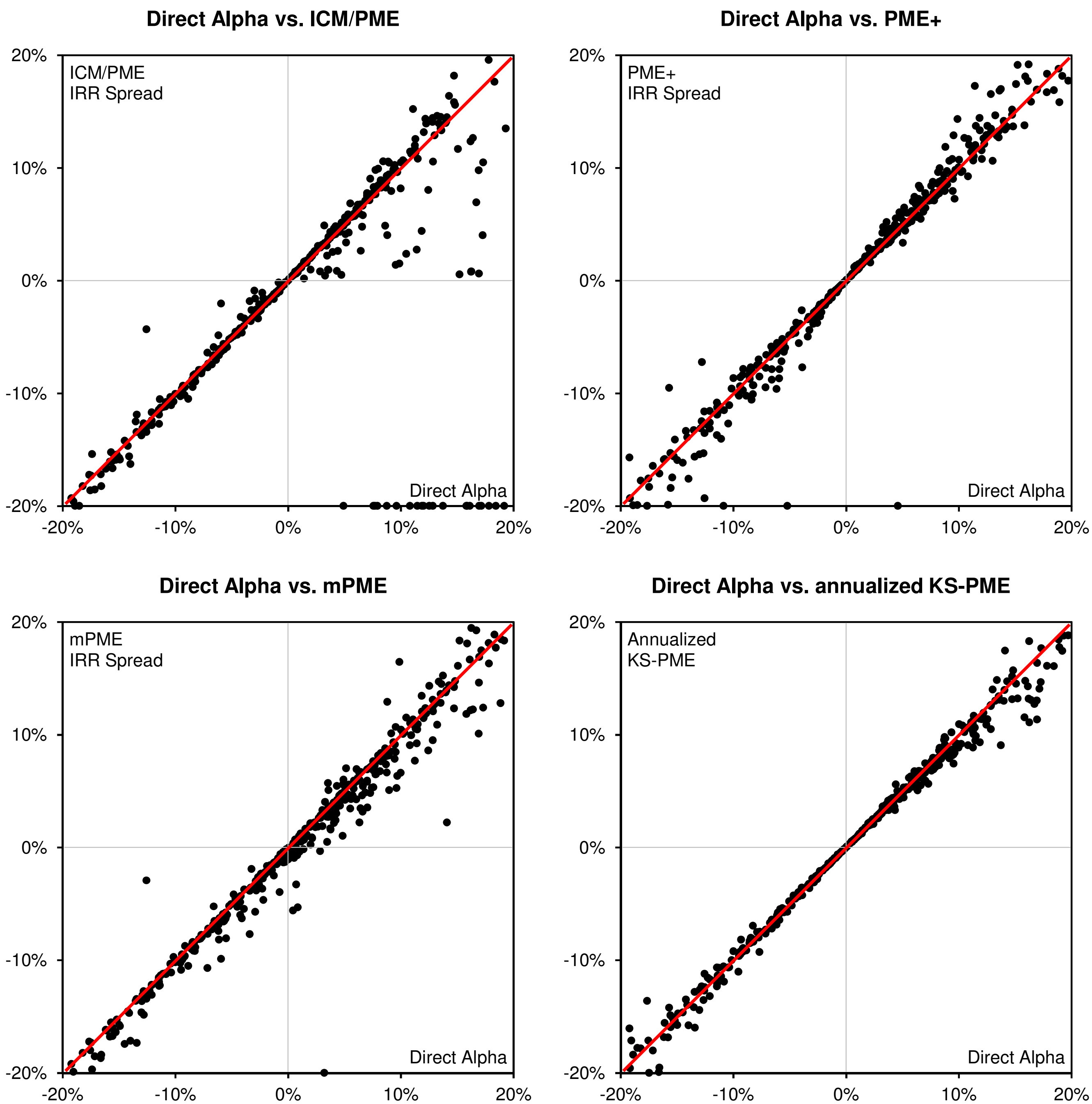

Figure 5: Comparison of Direct Alphas against different approximations

With respect to the IRR spread based on ICM/PME, the majority of observations show a reasonable fit with their corresponding Direct Alphas. However, there is some notable bias towards understating the exact excess return in the first quadrant, and vice versa for a small number of observations in quadrant three. ICM/PME IRRs that cannot be calculated due to a negative NAV_{ICM} (which needs to be balanced by a closing contribution) are shown with a spread of -20%. This is the case for about 5% of all funds.

The comparison against the PME+ IRR spread shows the well-known tendency of PME+ to inflate the IRR spread in case of higher returns by the PE fund compared to the public market, and vice versa. In addition, PME+ IRR spreads deviate significantly more from the exact rate of return relative to the S&P 500 than those of ICM/PME in most cases. In a couple of instances where the PME+ IRR could not be calculated, we have set the spread to -20%. This happens more often outside the [-20%, +20%] interval, though.

The mPME approach shows a similar fit for negative IRR spreads as ICM/PME, but a more pronounced dispersion of positive spreads. In addition, we observe a certain bias towards understating the rate of return relative to the S&P 500 in the first quadrant. Similarly to ICM/PME and PME+, a spread of -20% indicates the non-existence of an mPME IRR.

The annualized KS-PME is calculated as the n-th root of the regular KS-PME multiple (minus one), with n being the duration of the capital employed. This duration is based on the weighted-average date of all distributions minus the weighted-average date of all contributions of the PE fund. Overall, this approximation shows the closest fit with the exact alphas, yet some meaningful dispersion beyond the [-10%, +10%] intervals is present, too.

Conclusion

In recent years private equity practitioners have encountered a surprising problem: As several different methods of computing Public Market Equivalent (PME) returns have been introduced, it has remained unclear how the different methods are related and which methods might be reliable. In this white paper we have compared the Direct Alpha method with other approaches to calculating Public Market Equivalents.

As we have shown, Direct Alpha is the only method that produces the exact rate of return of outperformance as compared with a selected reference benchmark. The other methods examined here (ICM, PME+, and mPME) are all heuristic approximations that do not give the exact solution. Moreover, we have shown that all of the other PME methods are more complicated to compute than Direct Alpha, in addition to being less accurate.

We have also compared the results of the different methods over a large data set of cash flows from real funds. While many of the results are in fair agreement, all of the approximate heuristic methods have several cases that result in significant errors.

APPENDIX A

In this appendix we provide an outline of the derivation of our algorithm for estimating alpha in private equity. As we noted in the body of this white paper, alpha is that component of return that is systematically different from the return to market comparables. We start from the Capital Asset Pricing Model (CAPM) equation breaking out the components of return in excess of the risk-free rate:

$$r = \alpha + \beta m + \varepsilon \qquad \text{(Equation 1)}$$

The variables in this equation were defined in the body as follows:

- r is the (log) rate of return on the portfolio in question
- m is the (log) rate of return on the basket of market comparables
- β is a leverage factor
- α is the return due to skill or information
- ε is the random return due to chance

For this paper let's consider a simple case where there is no contribution from chance. Let's imagine a single contribution cashflow (we'll call it c_0) and a single distribution at a later time (c_n). We'll adopt the usual convention that contributions have negative sign, and distributions have positive sign. We'll also denote by B_t the value of a basket of comparables, developed through the due diligence process, at some time *t*. Then in the absence of any special skill or information we expect the contribution and distribution to be related by

$$c_n = -\left(\frac{B_n}{B_0}\right)c_0 \qquad \text{(Eq. 2)}$$

This is equivalent to saying that the log return to the investment, *r*, is equal to the log return to our benchmark, *b*:

$$r = b \qquad \text{(Eq. 3)}$$

Equation 3 looks a lot like the CAPM from Equation 1. In fact, the benchmark return to our comparables, *b*, can always be thought of as the appropriately levered return to some appropriate index, βm, that the CAPM calls for. It's just a question of how the index is defined.

So what if our investment has some additional alpha return due to manager information or skill? Then the return equation becomes

$$r = \alpha + b \qquad \text{(Eq. 4)}$$

When we solve Eq. 4 for the distribution in the presence of alpha we naturally find that it also has an additional factor:

$$c_n = -\exp(\alpha[t_n - t_0])\left(\frac{B_n}{B_0}\right)c_0 \qquad \text{(Eq. 5)}$$

Of course, realistic portfolios generally have a number of cash flows. Cash is additive, so if t_n is the final time and we add up the effect of a number of earlier cash flows as given in Eq. 5 we get

$$0 = \sum_{i=0}^{n} \exp(\alpha[t_n - t_i]) \left(\frac{B_n}{B_i} \right) c_i \qquad \text{(Eq. 6)}$$

Figure 4 in the body of this white paper is just a schematic representation of Eq. 6. The private equity cash flows are the c terms; the public benchmark values are the B terms; and α is what we want to solve for.

In this appendix we have not accounted for the specific volatility or the uncertainty in terminal value; the complete derivation, including these sources of uncertainty, can be found in (Griffiths, 2009). Chance makes the details a little more complicated, but it does not affect the concept. As you might imagine, the bigger the effect of chance, the harder it is to separate out the effects of skill. In statistical terms, we estimate the confidence interval around our estimate of alpha. As specific volatility decreases, and as the length of the data series increases, the confidence interval decreases. Thus, we have the best chance of identifying alpha in a long-term, well-diversified portfolio. However, if the magnitude of alpha creation is significant enough for less established portfolios or managers, a high confidence directional conclusion is also possible. When evaluating emerging managers and young investment programs, secondary market pricing can be used as a substitute for the current unrealized portfolio value in order to improve confidence in the alpha estimates. The details of this calculation can be found in (Griffiths, 2009).

Solving Equation 6 looks a little daunting, but we can simplify it by breaking up the various terms. First, we can define future-valued cash flows f_i, as suggested by Eq. 1:

$$f_i \equiv \left(\frac{B_n}{B_i} \right) c_i \qquad \text{(Eq. 7)}$$

We can see that f_i is the value we would expect to get at time t_n due to a cash flow at time t_i, assuming that the value of the investment evolves the same way as our benchmark portfolio.

Second, we can define the arithmetic excess rate of return a that is due to skill or information, corresponding to the log rate of return alpha. We use arithmetic rates of return when computing Internal Rate of Return, as we so often do in private equity:

$$1 + a \equiv \exp(\alpha) \qquad \text{(Eq. 8)}$$

Now we can use Eqs. 7 and 8 to rewrite Eq. 6, showing how the manager skill interacts with the values of the benchmark investments:

$$0 = \sum_{i=0}^{n} f_i (1 + a)^{(t_n - t_i)} \qquad \text{(Eq. 9)}$$

Thus we see that the actual calculation of alpha is just a simple internal rate of return (IRR) such as private equity investors compute every day.

From a computational point of view, then, the four input variables to calculate Direct Alpha are

- A series of contributions into the PE fund: $C = \{c_0, c_1, \dots, c_n\}$
- A series of distributions from the PE fund: $D = \{d_0, d_1, \dots, d_n\}$
- A residual value of the PE fund at time n: NAV_{PE}
- A reference benchmark (e.g., the S&P 500): $M = \{m_0, m_1, \dots, m_n\}$

Based on the series of contributions, distributions and benchmark values

- The future value of contributions at time n is: $FV(\mathrm{C}) = \left\{ \mathrm{c}_0 \cdot \frac{\mathrm{m}_n}{\mathrm{m}_0}, \mathrm{c}_1 \cdot \frac{\mathrm{m}_\mathrm{n}}{\mathrm{m}_1}, \dots, \mathrm{c}_\mathrm{n} \right\}$
- The future value of distributions at time n is: $FV(\mathrm{D}) = \left\{ \mathrm{d}_0 \cdot \frac{\mathrm{m}_n}{\mathrm{m}_0}, \mathrm{d}_1 \cdot \frac{\mathrm{m}_\mathrm{n}}{\mathrm{m}_1}, \dots, \mathrm{d}_\mathrm{n} \right\}$

The arithmetic (or discrete-time) Direct Alpha is then calculated via the IRR over the series of net cash flows of the future values of contributions, distributions and the final $\mathrm{NAV}_{\mathrm{PE}}$

$$\mathrm{a} = IRR(FV(C), FV(D), NAV_{PE})$$

The continuous-time Direct Alpha is

$$\mathrm{alpha} = \frac{\ln(1 + \mathrm{a})}{\Delta}$$

where Δ is the time interval for which alpha is computed (typically one year).

In case of using the present value perspective

- The present value of contributions at time 0 is: $PV(\mathrm{C}) = \left\{ \mathrm{c}_0, \mathrm{c}_1 \cdot \frac{\mathrm{m}_0}{\mathrm{m}_1}, \dots, \mathrm{c}_\mathrm{n} \cdot \frac{\mathrm{m}_0}{\mathrm{m}_n} \right\}$
- The present value of distributions at time 0 is: $PV(\mathrm{D}) = \left\{ \mathrm{d}_0, \mathrm{d}_1 \cdot \frac{\mathrm{m}_0}{\mathrm{m}_1}, \dots, \mathrm{d}_\mathrm{n} \cdot \frac{\mathrm{m}_0}{\mathrm{m}_n} \right\}$
- The present value of the $\mathrm{NAV}_{\mathrm{PE}}$ at time 0 is: $PV(NAV_{PE}) = NAV_{PE} \cdot \frac{\mathrm{m}_0}{\mathrm{m}_n}$

The arithmetic (or discrete-time) Direct Alpha is

$$\mathrm{a} = IRR\big(PV(C), PV(D), PV(NAV_{PE})\big)$$

A full derivation of Direct Alpha can be found in Griffiths (2009), and Gredil, Griffiths and Stucke (2014).

APPENDIX B

When studying the literature on private equity (PE) benchmarking, the interested reader immediately witnesses a large number of different approaches, each of which trying to estimate the excess return that private equity has generated against public markets. We briefly point at the four most common approaches in the following. A detailed description and examples can be found in Gredil, Griffiths, and Stucke (2014).

The journey starts in 1992, when Long and Nickels (1995) first introduced their ICM approach. ICM, also known as public market equivalent (PME), invests and divests a PE fund's cash flows with a public equity index to arrive at the terminal value that public equities would have generated instead. Comparing the PE IRR with the ICM/PME IRR, which is derived from the same series of cash flows and the different NAV_{ICM}, yields a positive IRR spread if PE has delivered higher returns than the index, and vice versa. More formally, this can be expressed as

$$NAV_{ICM} = \sum FV(C) - \sum FV(D) \;\; => \;\; IRR_{ICM} = IRR(C, D, NAV_{ICM}) \;\; => \;\; IRR\ Spread = IRR_{PE} - IRR_{ICM}$$

Although deeply appealing, the concept of different terminal values has various disadvantages that may lead to unreliable results (see Figure 5).

In 2002, Kaplan and Schoar (2005) first introduced their own PME approach, commonly referred to as KS-PME, which is a multiple measuring the excess wealth from a PE fund over public equities. In line with ICM/PME contributions into the PE fund are invested in the index. Distributions by the PE fund, however, are not withdrawn from the index investments but reinvested in the index instead. The KS-PME is then calculated as the ratio of this future value of distributions (plus a potential residual value of the fund) over the future value of all contributions. A ratio above one indicates excess wealth generated by the PE fund, and vice versa. More formally, this can be expressed as

$$KS - PME = (\sum FV(D) + NAV_{PE}) / \sum FV(C) \;\; \text{or} \;\; KS - PME = TVPI(FV(C), FV(D), NAV_{PE})$$

While this approach is a formally correct comparison and yields robust results, the disadvantage is that it does not provide information on the rate at which this excess wealth has been generated.

In 2003, Rouvinez (2003) and Capital Dynamics introduced the PME+ approach.[2] To avoid the issues of different NAVs by ICM/PME, PME+ rescales all distributions from the public equity investments before calculating their PME+ IRR, such that both terminal values are the same at the time of the analysis (or zero if the PE fund has been liquidated). More formally, this can be expressed as

$$s = (\sum FV(C) - NAV_{PE}) / \sum FV(D) \;\; => \;\; IRR_{PME+} = IRR(C, s \cdot D, NAV_{PE}) \;\; => \;\; IRR\ Spread = IRR_{PE} - IRR_{PME+}$$

The main disadvantage of PME+ is that the IRR measure is very sensitive to early distributions, and rescaling them inevitably inflates or deflates the resulting IRR spread.

Since the late-2000s, Cambridge Associates uses the mPME approach. Instead of using a fixed scaling factor like PME+, the mPME approach rescales all distributions by a time-varying factor. In essence, distributions by the PE fund are no longer matched in absolute capital terms against the index investments, but proportionately with respect to the ratio of the succeeding interim value of the index investments and the succeeding interim NAV by the fund. While fair in spirit, the disadvantages of mPME are similar to those of PME+. However, using a time-varying scaling factor may cause an additional problem: even if the public and private investments have exactly the same true returns, mPME will return

[2] Note that Capital Dynamics has been granted a U.S. patent for PME+ in 2010 (#7,698,196).

different results if there are any pricing errors in the interim NAVs of the PE fund. A more formal expression of the mPME approach can be found in Gredil, Griffiths and Stucke (2014).

Figure 6 provides an overview of the advantages and weaknesses of the different PME approaches.

Leaving aside the individual issues of ICM/PME, PME+ and mPME, the main problem is that they cannot – by definition – arrive at a PE fund's exact rate of return relative to the benchmark. All three approaches follow an *indirect* way by calculating the IRR of a hypothetical public equity portfolio first, and then infer the spread against the PE fund's IRR. The non-additive nature of compound rates, however, effectively prevents the calculation of the exact alpha in an *indirect* way (see Cauchy's functional equation). As a result, existing PME approaches represent heuristic approximations by definition.

Method	Authors	Advantages	Weaknesses
Index Comparison Method (ICM), a.k.a. Public Market Equivalent (PME)	Long, Nickels	• Intuitive approach	• IRR spread sensitive to terminal value and fund age • IRR spread may be biased • Not always defined • No exact solution
Kaplan/Schoar Public Market Equivalent (PME_{KS})	Kaplan, Schoar	• Formally correct method • Always defined	• Not an annualized measure
Public Market Equivalent Plus (PME+)	Rouvinez	• Identical residual values • Liquidating reference portfolio	• Inflated/deflated IRR spreads • Not always defined • No exact solution
Modified Public Market Equivalent (mPME)	Cambridge Associates	• Liquidating reference portfolio	• Inflated/deflated IRR spreads • Sensitive to pricing errors • No exact solution

Figure 6: Advantages and weaknesses of the different PME approaches

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Dr. Rüdiger Stucke is the SL Capital Partners Research Fellow in Private Equity at the Saïd Business School, Oxford University. Dr. Stucke received his PhD in Finance with summa cum laude from Paderborn University, Germany. Previously he graduated with summa cum laude in Business Administration, Economics and Computer Science (BSc and MSc).

Dr. Stucke is also a Landmark Fellow. The Landmark Fellows program is intended to provide insight on current academic research in private equity for Landmark Partners and its investors. Landmark provides the Landmark Fellows with a stipend for these services.

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